

**A-Level Mathematics**

**Paper 5**

**Unsolved Topical**

**Past Papers with Examiner Report**

**All Variants**

**2014-2021**

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## **PREFACE**

Excellence in learning cannot be claimed without application of concepts in a dexterous way. In this regard one of the logical approach is to start in chunks; like chapter wise learning and applying the concept on exam based questions.

This booklet provides an opportunity to candidates to practice topic wise questions from previous years to the latest. Extensive working of Team MS Books has tried to take this booklet to perfection by collaborating with top of the line teachers.

We have added answer key / marks scheme at the end of each topic for the candidate to compare the his/her answer to the best.

MS Books strives to maintain actual spacing between consecutive questions and within options as per CAIE format which gives students a more realistic feel of attempting question.

Review, feedback and contribution in this booklet by various competent teachers of a subject belonging to renowned school chains make it most valuable resource and tool for both teachers and students.

With all belief in strength of this resource material I can confidently claim that it is worth in achieving brilliance.

Our sincere thanks and gratification to Mr. Zafar Iqbal who took out special time to help compile and manage this booklet. We would also like to appreciate Mathematics faculty for reviewing and indorsing it.

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## Representation of Data

Q7/61/M/J/14

- 1 A typing test is taken by 111 people. The numbers of typing errors they make in the test are summarised in the table below.

|                         |       |        |         |         |         |
|-------------------------|-------|--------|---------|---------|---------|
| Number of typing errors | 1 – 5 | 6 – 20 | 21 – 35 | 36 – 60 | 61 – 80 |
| Frequency               | 24    | 9      | 21      | 15      | 42      |

- (i) Draw a histogram on graph paper to represent this information. [5]
- (ii) Calculate an estimate of the mean number of typing errors for these 111 people. [3]
- (iii) State which class contains the lower quartile and which class contains the upper quartile. Hence find the least possible value of the interquartile range. [3]

Q6/62/M/J/14

- 2 The times taken by 57 athletes to run 100 metres are summarised in the following cumulative frequency table.

|                      |        |        |        |        |        |        |
|----------------------|--------|--------|--------|--------|--------|--------|
| Time (seconds)       | < 10.0 | < 10.5 | < 11.0 | < 12.0 | < 12.5 | < 13.5 |
| Cumulative frequency | 0      | 4      | 10     | 40     | 49     | 57     |

- (i) State how many athletes ran 100 metres in a time between 10.5 and 11.0 seconds. [1]
- (ii) Draw a histogram on graph paper to represent the times taken by these athletes to run 100 metres. [4]
- (iii) Calculate estimates of the mean and variance of the times taken by these athletes. [4]

Q1/63/M/J/14

- 3 Some adults and some children each tried to estimate, without using a watch, the number of seconds that had elapsed in a fixed time-interval. Their estimates are shown below.

Adults: 55 58 67 74 63 61 63 71 56 53 54 78 73 64 62  
 Children: 86 95 89 72 61 84 77 92 81 54 43 68 62 67 83

- (i) Draw a back-to-back stem-and-leaf diagram to represent the data. [3]
- (ii) Make two comparisons between the estimates of the adults and the children. [2]

Q4/63/M/J/14

- 4 The heights,  $x$  cm, of a group of 28 people were measured. The mean height was found to be 172.6 cm and the standard deviation was found to be 4.58 cm. A person whose height was 161.8 cm left the group.

- (i) Find the mean height of the remaining group of 27 people. [2]
- (ii) Find  $\sum x^2$  for the original group of 28 people. Hence find the standard deviation of the heights of the remaining group of 27 people. [4]

Q1/61/O/N/14

- 5 Find the mean and variance of the following data. [3]

5    -2    12    7    -3    2    -6    4    0    8

Q4/61/O/N/14

- 6 The following back-to-back stem-and-leaf diagram shows the times to load an application on 61 smartphones of type *A* and 43 smartphones of type *B*.

|      | Type A                    |   | Type B                  |      |
|------|---------------------------|---|-------------------------|------|
| (7)  | 9 7 6 6 4 3 3             | 2 | 1 3 5 8                 | (4)  |
| (7)  | 5 5 4 4 2 2 2             | 3 | 0 4 4 5 6 6 6 6 7 8 8 9 | (12) |
| (13) | 9 9 8 8 8 7 6 6 4 3 2 2 0 | 4 | 0 1 1 2 3 6 8 8 9 9     | (10) |
| (9)  | 6 5 5 4 3 2 1 1 0         | 5 | 2 5 6 6 9               | (5)  |
| (4)  | 9 7 3 0                   | 6 | 1 3 8 9                 | (4)  |
| (6)  | 8 7 4 4 1 0               | 7 | 5 7                     | (2)  |
| (10) | 7 6 6 6 5 3 3 2 1 0       | 8 | 1 2 4 4                 | (4)  |
| (5)  | 8 6 5 5 5                 | 9 | 0 6                     | (2)  |

Key: 3 | 2 | 1 means 0.23 seconds for type *A* and 0.21 seconds for type *B*.

- (i) Find the median and quartiles for smartphones of type *A*. [3]

You are given that the median, lower quartile and upper quartile for smartphones of type *B* are 0.46 seconds, 0.36 seconds and 0.63 seconds respectively.

- (ii) Represent the data by drawing a pair of box-and-whisker plots in a single diagram on graph paper. [3]

- (iii) Compare the loading times for these two types of smartphone. [1]

Q6/62/O/N/14

- 7 On a certain day in spring, the heights of 200 daffodils are measured, correct to the nearest centimetre. The frequency distribution is given below.

| Height (cm) | 4 – 10 | 11 – 15 | 16 – 20 | 21 – 25 | 26 – 30 |
|-------------|--------|---------|---------|---------|---------|
| Frequency   | 22     | 32      | 78      | 40      | 28      |

- (i) Draw a cumulative frequency graph to illustrate the data. [4]
- (ii) 28% of these daffodils are of height  $h$  cm or more. Estimate  $h$ . [2]
- (iii) You are given that the estimate of the mean height of these daffodils, calculated from the table, is 18.39 cm. Calculate an estimate of the standard deviation of the heights of these daffodils. [3]

Q2/63/O/N/14

- 8 A traffic camera measured the speeds,  $x$  kilometres per hour, of 8 cars travelling along a certain street, with the following results.

62.7 59.6 64.2 61.5 68.3 66.9 62.0 62.3

- (i) Find  $\Sigma(x - 62)$ . [1]
- (ii) Find  $\Sigma(x - 62)^2$ . [1]
- (iii) Find the mean and variance of the speeds of the 8 cars. [3]

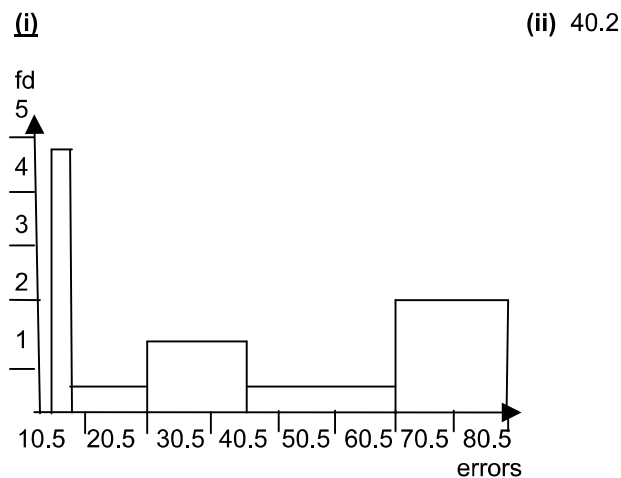
Q7/61/M/J/14

**Question 1**

Surprisingly, the majority of candidates scored less than half marks on this basic question on grouped data.

- (i) Very few Centres were familiar with 'frequency density' and most candidates produced a frequency histogram or a cumulative frequency curve instead. The class boundaries of 0.5, 5.5, 20.5, etc were often not realised and marks were lost on the graph through untidy presentation. A line across the top of a rectangle indicating the height (value) gets no credit if it is not horizontal or several mm thick.
- (ii) A very large minority of candidates did not use the 'mid class values', preferring to use wrongly the class widths or various class boundaries instead.
- (iii) Frequently 27.75 and 83.25 (or 28 and 84) were given as the quartile values and the question of which class was ignored.

Answers:



(iii) (6-20) Class, (61-80) Class, 41.

Q6/62/M/J/14

**Question 2**

Part (i) should have been very straightforward but many candidates failed to read the question carefully and were unable to score the mark. Part (i) should have helped candidates to realise that frequencies were needed before they could find frequency densities to draw the histogram in part (ii). Generally the graph plotting was not good, with candidates drawing lines freehand, poor scales going up in 3s, no labelling of axes.

Answers: 6, 11.7, 0.547

Q1/63/M/J/14

**Question 3**

Most candidates understood the concept of a stem-and-leaf diagram.

- (i) Good solutions had the data in order and the terms evenly spaced. Better candidates initially produced an unordered stem-and-leaf diagram. Many did not realise that the diagram enabled the distributions to be compared by the shape produced. Most candidates provided two separate keys for the diagram rather than the required single back-to-back key. Good candidates included the units that were required for the correct interpretation of the key.
- (ii) Most candidates made comparisons about the range and median of the data, although weaker candidates made statements without a comparison. A number of candidates commented on the shape of the distribution. A significant number of candidates commented twice on a single aspect of the data.

Answer: (ii) children's estimates more spread out, adults' estimates lower

Q4/63/M/J/14

**Question 4**

Many candidates gained full credit for this question. Good solutions showed clear algebraic manipulation throughout.

- (i) Almost all candidates attempted this question successfully. A few candidates did not use the correct number of people in the new group.
- (ii) Most candidates rearranged the standard deviation formula accurately to find  $\Sigma x^2$  of the original group. Many candidates stated this to 3 significant figures. Good solutions then used their accurate answer to calculate the removal of the person identified in (i) before using the formula again. Weaker candidates often used either their rounded interim answer or failed to remove the additional person before calculating.

Answers: (i) 173 (ii)  $\Sigma x^2 = 834700$ ,  $\sigma = 4.16$

Q1/61/O/N/14

**Question 5**

This was a routine question on finding the mean and variance from a set of 10 discrete data points. Nearly all candidates gained full marks on this question apart from the few who thought that  $(-2)^2 = -4$ .

Answers: 2.7, 27.8

Q4/61/O/N/14

**Question 6**

This was a straightforward question for those who knew their descriptive statistics: finding the median and quartiles and drawing a box-and-whisker plot of the data. Unfortunately many candidates did not calculate the median using  $\frac{(n+1)}{2}$  or the quartiles as  $\frac{(n+1)}{4}$  and  $(n+1) \times \frac{3}{4}$ , losing a couple of marks. However, follow-through marks meant that candidates could still gain 3 marks by correctly drawing the axis and labelling it, and drawing the two box-and-whisker graphs. Very few candidates labelled their axis and thus most lost a mark. The comments at the end were good.

Answers: LQ 0.41, Q2 0.52, UQ 0.79, smartphone B is quicker, slightly less variable etc.

Q6/62/O/N/14

**Question 7**

This question on descriptive statistics was not done well. Teachers need to stress the importance of labels, accurate graph drawing, and sensible scales. Candidates who choose a scale of 10 small squares to represent 25 units or 30 units or 33 units, in order to fill up the page entirely, are not going to gain full marks for accurate plotting of points. Part (ii) was the worst-attempted question part in the entire paper, with only about 5% of candidates realising that 28% of daffodils had a height of  $h$  or more meant that 72% had a height of  $h$  or less, and that is what a cumulative frequency graph shows. Part (iii) was routinely done and many scored full marks though some used class boundaries or lower or upper end points when calculating the mean.

Answers: (ii) a single value between 21 and 23 cm, 6.01

Q2/63/O/N/14

**Question 8**

Many candidates did not understand the requirements of this question, which was based upon an 'assumed mean' and then calculating the appropriate mean and variance. A surprising number of simple numerical errors were identified within calculations. Many candidates did not work with at least 4 significant figures throughout their solutions, and so lost accuracy marks.

- (i) Good candidates created a data table which enabled accurate calculation of the individual values required for all parts of this question. Many candidates misinterpreted the expression and calculated  $\Sigma x - 62$ . A few candidates calculated the variance at this stage and then attempted to work back to the required value.



- (ii) Few correct solutions were seen for this part, with the best methods linking with the table created for part (i). Good candidates stated the individual terms which were to be evaluated. However, most candidates interpreted the expression as requiring their value to part (i) to be squared. A number of candidates attempted to expand the expression and then were unable to evaluate the resulting expression.
- (iii) Most candidates successfully completed this part of the question. This was due to almost all candidates using the correct formulae with the original data, rather than using their answers from parts (i) and (ii). Some candidates who had previously calculated the mean or variance then recalculated at this stage, sometimes with a different result.

Answer: (i) 11.5 (ii) 75.1 (iii) 63.4, 7.32

Q4/63/O/N/14

### Question 9

Candidates of all abilities attempted this question with some success. However, few fully correct solutions were seen.

- (i) Good solutions had the data in order and the terms evenly spaced. Better candidates initially produced an unordered stem-and-leaf diagram. Good candidates included the units that were required for the correct interpretation of the key. Many candidates did not show that they understood the necessity for the 'leaf' to be consistently spaced, with corrections often making the leaf inaccurate. Although most candidates recognised that there should be a key, few included the relevant context unit of 'glasses'.
- (ii) The best solutions had the relevant critical values clearly identified before any attempt to draw the box-and-whisker plot on graph paper. Most candidates identified the median correctly using  $\frac{(n+1)}{2}$  but many then did not use  $\frac{(n+1)}{4}$  and  $(n+1) \times \frac{3}{4}$  to identify the quartiles. The majority of candidates chose an appropriate scale to plot their diagram, with 2 cm to 10 glasses being the most accurately used. Few candidates continued the 'whiskers' through the box. A small number of candidates calculated and plotted 'outliers' which is outside the requirements of the syllabus. Few candidates plotted a linear scale with contextual labelling. The solutions of the weakest candidates frequently neither stated the critical values nor had any scale on their graph.

Q5/61/M/J/15

### Question 10

- (i) Few candidates understood the basic principle of combining the 27 weights to obtain their mean.
- (ii) Many good attempts were marred by omitting  $\bar{x}^2$ , not multiplying by 9 or 18 throughout and losing the final mark by using 5.83 instead of 5.833 or better.

Answers: (i) 5.83 (ii) 1.46

Q2/62/M/J/15

### Question 11

A surprising number of candidates did not realise that mid-points have to be used. Frequently class widths, upper class boundaries lower class boundaries were used. Once again, the final mark was lost by many candidates through not working with 4 figures. They used 45.8 when subtracting  $\bar{x}^2$  whereas 45.83 would have given the correct answer to 3sf.

Answers: 45.8, 14.9