A-Level Mathematics

Paper 3

Unsolved Topical

Past Papers with Marking Schemes

All Variants

2014-2021

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PREFACE

Excellence in learning cannot be claimed without application of concepts in a dexterous way. In this regard one of the logical approach is to start in chunks; like chapter wise learning and applying the concept on exam based questions.

This booklet provides an opportunity to candidates to practice topic wise questions from previous years to the latest. Extensive working of Team MS Books has tried to take this booklet to perfection by collaborating with top of the line teachers.

We have added answer key / marks scheme at the end of each topic for the candidate to compare the his/her answer to the best.

MS Books strives to maintain actual spacing between consecutive questions and within options as per CAIE format which gives students a more realistic feel of attempting question.

Review, feedback and contribution in this booklet by various competent teachers of a subject belonging to renowned school chains make it most valuable resource and tool for both teachers and students.
With all belief in strength of this resource material I can confidently claim that it is worth in achieving brilliance.

Our sincere thanks and gratification to Mr. Zafar Iqbal who took out special time to help compile and manage this booklet. We would also like to appreciate Mathematics faculty for reviewing and indorsing it.

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TABLE OF CONTENTS

Algebra

Q9/31/M/J/14

1 **(i)** Express $\dfrac{4+12x+x^2}{(3-x)(1+2x)^2}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{4+12x+x^2}{(3-x)(1+2x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]

Q1/32/M/J/14

2 Find the set of values of x satisfying the inequality

$$|x+2a| > 3|x-a|,$$

where a is a positive constant. [4]

Q5/32/M/J/14

3 **(i)** The polynomial $f(x)$ is of the form $(x-2)^2 g(x)$, where $g(x)$ is another polynomial. Show that $(x-2)$ is a factor of $f'(x)$. [2]

(ii) The polynomial $x^5 + ax^4 + 3x^3 + bx^2 + a$, where a and b are constants, has a factor $(x-2)^2$. Using the factor theorem and the result of part **(i)**, or otherwise, find the values of a and b. [5]

Q2/33/M/J/14

4 Expand $(1+3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

Q3/32/O/N/14

5 The polynomial $ax^3 + bx^2 + x + 3$, where a and b are constants, is denoted by $p(x)$. It is given that $(3x+1)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x-2)$ the remainder is 21. Find the values of a and b. [5]

Q9/32/O/N/14

6 Let $f(x) = \dfrac{x^2-8x+9}{(1-x)(2-x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Q1/33/O/N/14

7 Solve the inequality $|3x-1| < |2x+5|$. [4]

Q3/33/O/N/14

8 The polynomial $4x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(x+1)$ and $(x+2)$ are factors of $p(x)$.

(i) Find the values of a and b. [4]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by (x^2+1). [3]

Q3/31/M/J/15

9 Show that, for small values of x^2,

$$(1-2x^2)^{-2}-(1+6x^2)^{\frac{2}{3}} \approx kx^4,$$

where the value of the constant k is to be determined. [6]

Q8/32/M/J/15

10 Let $f(x) = \dfrac{5x^2+x+6}{(3-2x)(x^2+4)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Q2/33/M/J/15

11 Solve the inequality $|x-2| > 2x-3$. [4]

Q1/32/O/N/15

12 Solve the inequality $|2x-5| > 3|2x+1|$. [4]

Q6/32/O/N/15

13 The polynomial $8x^3+ax^2+bx-1$, where a and b are constants, is denoted by $p(x)$. It is given that $(x+1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x+1)$ the remainder is 1.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $p(x)$ completely. [3]

Q2/33/O/N/15

14 Given that $\sqrt[3]{(1+9x)} \approx 1+3x+ax^2+bx^3$ for small values of x, find the values of the coefficients a and b. [3]

Q1/31/M/J/16

15 **(i)** Solve the equation $2|x-1| = 3|x|$. [3]

(ii) Hence solve the equation $2|5^x-1| = 3|5^x|$, giving your answer correct to 3 significant figures. [2]

Q8/31/M/J/16

16 Let $f(x) = \dfrac{4x^2+12}{(x+1)(x-3)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

Q2/32/M/J/16

17 Expand $\dfrac{1}{\sqrt{(1-2x)}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

Q9/31/M/J/14

Question 1

(i) Usually one of the correct forms for the partial fractions was chosen. Despite several candidates making some arithmetical errors in establishing the coefficients of the terms in the partial fractions this section contained some of the best work from the candidates.

(ii) Whilst most candidates knew how to perform a binomial expansion once they had an expression of the form $(1-\frac{x}{3})^{-1}$, $(1+2x)^{-1}$ or $(1+2x)^{-2}$, many did not reach these forms as errors were introduced as the constants 3 and 2 were nearly always incorrectly brought into the numerator.

Answers: **(i)** $\frac{1}{3-x}+\frac{3}{2(1+2x)}-\frac{1}{2(1+2x)^2}$ or $\frac{1}{3-x}+\frac{3x+1}{(1+2x)^2}$ **(ii)** $\frac{4}{3}-\frac{8}{9}x+\frac{1}{27}x^2$

Q1/32/M/J/14

Question 2

This was only fairly well answered. Candidates need to be secure in working on literal equations and inequalities. The presence of literal constants seemed to unsettle many candidates. Rather than obtain the critical values by solving the simple linear equations $(x + 2a) = \pm 3(x - a)$, the majority squared the given inequality and then tried to solve a non-modular quadratic equation or inequality. Apart from the error of failing to square the factor of 3, mistakes in handling the literal coefficients and constants were frequent and some gave a value to the variable *a*.

Answer: $\frac{1}{4}a < x < \frac{5}{2}a$

Q5/32/M/J/14

Question 3

The response to part **(i)** was disappointing. Only a minority obtained a correct derivative of f(x). Expressions such as $2(x - 2)g(x)$ or $2(x - 2)g'(x)$ were much more common. Of those with a correct derivative nearly all went on to factorise it or to show that f′(2) = 0.

In part **(ii)**, many started by substituting $x = 2$ in the given polynomial and obtaining a correct equation in the unknowns. Those who applied the result of part **(i)**, as suggested in the wording of the question, easily obtained a second equation and completed the problem quickly. Failure to differentiate the constant term of the polynomial was the commonest error here. However a large number of time consuming attempts were seen which involved identifying the polynomial with $(x - 2)^2 (Ax^3 + Bx^2 + Cx + D)$, $(x^2 - 4x + 4)$. Only a few of these attempts sustained accuracy for long enough to make useful progress.

Answer: **(ii)** $a = -4$, $b = 3$

Q2/33/M/J/14

Question 4

Again many correct answers were seen, however a few candidates made an error in simplifying one of the coefficients. Multiplying the final answer by a factor of 3 to clear the denominator was penalised by the loss of one accuracy mark.

Answer: $1 - x + 2x^2 - \frac{14}{3}x^3$

Q3/32/O/N/14

Question 5

This was well answered overall. The factor and remainder theorems were used appropriately by most candidates and marks were only lost through the occasional slip in the algebra. Those who tried to form the equations in *a* and *b* by long division rarely completed the question correctly.

Answer: $a = 12$, $b = -20$

Q9/32/O/N/14

Question 6

Part **(i)** was very well answered. A minority chose to work with an inappropriate form of partial fractions, but most solutions were either fully correct or evaluated all but one constant correctly. When setting up the preliminary identity involving the numerator $x^2 - 8x + 9$, candidates should check carefully that they have not miscopied the numerator and that they have multiplied the numerator of each partial fraction by the correct factor or combination of factors. The unwanted presence of an extra factor of $(2 - x)$ in the identity was a regular source of error. In part **(ii)** most could state and use a correct expansion of $(1-x)^{-1}$. They were less successful with $(2-x)^{-1}$ and $(2-x)^{-2}$. Errors such as taking $(2-x)^{-1}$ to be equivalent to $2(1-\frac{1}{2}x)^{-1}$ or $\frac{1}{2}(1-x)^{-1}$ were quite frequently encountered. As a result there were errors in most solutions.

Answers: **(i)** $\frac{2}{(1-x)} - \frac{1}{(2-x)} + \frac{3}{(2-x)^2}$, or $\frac{2}{(1-x)} + \frac{x+1}{(2-x)^2}$ **(ii)** $\frac{9}{4} + \frac{5}{2}x + \frac{39}{16}x^2$

Q1/33/O/N/14

Question 7

The majority of candidates preferred to form a quadratic inequality by squaring both sides of the modular inequality. This approach was usually successful, but candidates who made a slip in their algebra and reached an incorrect quadratic inequality often scored no further mark because they used their calculator to solve it and showed no working. They should be aware that method marks cannot be awarded when the answer is incorrect and no method is shown. A good number of candidates opted to draw a sketch and solve linear inequalities, usually very successfully.

Answer: $-\frac{4}{5} < x < 6$

Q3/33/O/N/14

Question 8

There were many fully correct solutions to part **(i)**. The most concise solutions were usually from candidates who started by substituting $x = -1$ and $x = -2$ and then went on to solve the resulting simultaneous equations. Those who tried to expand three brackets sometimes made slips in the working. In part **(ii)** most candidates set out to use algebraic division, but dividing by a quadratic factor often caused problems with terms not lined up correctly and then slipping into the incorrect columns. A few candidates were not expecting a linear remainder and tried to continue with the division, resulting in a quotient involving algebraic fractions. Similarly, some candidates who used the method of undetermined coefficients did not allow for a linear remainder. A small number of candidates confused the quotient with the remainder. Some candidates tried to find the remainder when using a complex linear factor $x \pm i$. They often reached a correct term, $i - 13$, but rarely related this back to the value of x substituted.

Answers: **(i)** $a = 11$, $b = 5$ **(ii)** $x - 13$

Q3/31/M/J/15

Question 9

Many candidates decided to express the first term in the expression as $\frac{1}{(1-2x^2)^2}$ and to then expand this denominator, hence making no progress. The positive index of the second term avoided this problem; however it was still very common to see the omission of negative signs and x^2, or even x, instead of $6x^2$ raised to a power.

Answer: 16

Q8/32/M/J/15

Question 10

Part **(i)** was very well answered. Almost all stated a correct form of partial fractions such as $\frac{A}{3-2x}+\frac{Bx+C}{x^2+4}$ and most solutions were either fully correct or evaluated all but one constant correctly. In part **(ii)** most had a good idea of the overall method but the fact that $B = -1$ and $C = -2$ caused problems for some. For example, they took the numerator of the second partial fraction to be $x - 2$. In forming the expansions, errors such as taking $(3-2x)^{-1}$ to be equivalent to $3(1-\frac{2}{3}x)^{-1}$ or $(x^2+4)^{-1}$ to be equivalent to $4(1+\frac{1}{4}x^2)^{-1}$ were quite frequently encountered.

Answers: **(i)** $\frac{3}{3-2x}-\frac{x+2}{x^2+4}$; **(ii)** $\frac{1}{2}+\frac{5}{12}x+\frac{41}{72}x^2$

Q2/33/M/J/15

Question 11

This question required the solution of a modular inequality. The candidate needed to (i) remove the modulus, (ii) obtain a critical value, here $\frac{5}{3}$, and (iii) interpret the value to obtain the appropriate solution. Those candidates who first sketched the graphs almost always scored full marks. However, the majority of candidates adopted a purely algebraic approach either by squaring both sides of the inequality or by solving two linear inequalities. Only a minority of candidates who approached the problem in this purely algebraic fashion were able to obtain a complete solution. The method of squaring both sides of the inequality to remove the modulus sign is not the best approach for this type of inequality since it introduces the additional critical point $x = 1$ and leaves the candidate with the problem of interpreting the region in which the original modular inequality is satisfied.

Answer: $nx < \frac{5}{3}$

Q1/32/O/N/15

Question 12

Considering that this is a standard piece of work it was not always done particularly well. Most candidates attempted to obtain a non-modular equation by squaring both sides. Errors such as failing to square the factor of 3 or taking the modulus of, say, $2x+1$ to be $\sqrt{(2x+1)}$ or $(2x+1)^2$ were not infrequent. Having squared both sides, candidates reduced the problem to a 3-term quadratic inequality or equation. This involved finding at least 9 coefficients and errors were common. Very few considered the initial expression as a difference of two squares so that the inequality becomes $(2x-5-(6x+3))(2x-5+(6x+3))>0$. It then follows that $(-4x-8)(8x-2)>0$, from which the critical values and solution are easily found. The few attempts using graphs were usually successful as were those working with a pair of linear equations. Those that set out with linear inequalities, almost always omitting the ranges for which they were valid, usually found the critical values but were often unable to derive the correct final answer.

Answer: $-2 < x < \frac{1}{4}$

Q6/32/O/N/15

Question 13

(i) Candidates usually applied the factor and remainder theorems correctly and obtained the two constants. A few equated $p(-\frac{1}{2})$ to 0, or changed 1 to 0 in the course of the working.

(ii) Whatever the values of *a* and *b*, most started the division by (x + 1) correctly but some attempted division by ($2x$ + 1). A few candidates with correct working did not factorise the quotient $8x^2 - 2x - 1$.

Answers: **(i)** $a = 6$, $b = -3$ **(ii)** $(x+1)(4x+1)(2x-1)$

Q2/33/O/N/15

Question 14

Many candidates demonstrated a good understanding of how to use the binomial expansion for a rational index. Most solutions started with a correct unsimplified expansion as far as the term in x^3, but there were several slips in the arithmetic in reaching the final answers. The most common error in the expansion was to use powers of *x* rather than of 9*x*. A small number of candidates attempted to find the cube of the right hand side and compare coefficients rather than start by expressing the cube root in index form.

Answer: $a = -9$, $b = 45$

Q1/31/M/J/16

Question 15

(i) Some candidates found one root by inspection (usually $x = -2$) and did not attempt to find the second root. Those candidates who started by forming a quadratic equation were usually successful in reaching both roots, provided they started with $4(x-1)^2 = 9x^2$ and not the popular incorrect form $2(x-1)^2 = 3x^2$.

(ii) Only a small number of candidates made use of the link between the two parts of this question and went directly to a solution of $5^x = \frac{2}{5}$. The majority of candidates started again from the beginning, sometimes starting with incorrect equations in logarithms. Some candidates attempted to solve $5^x = -2$ by first taking out the negative and then replacing it in their final answer.

Answers: **(i)** -2, $\frac{2}{5}$ **(ii)** -0.569

Q8/31/M/J/16

Question 16

(i) The majority of candidates split the fraction correctly into the form $\frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$ but the alternative form $\frac{A}{x+1} + \frac{Bx+C}{(x-3)^2}$ was also accepted. Many candidates are very proficient in completing this process, with some showing very little working to support their answers. Candidates should understand that in showing no working, if the process goes wrong they will earn no credit for having a correct method.

(ii) Many candidates gave a correct expansion of $(1+x)^{-1}$. The other two terms were more demanding, requiring the steps $(x-3)^{-1} = (-3)^{-1}\left(1-\frac{x}{3}\right)^{-1}$ and $(x-3)^{-2} = (-3)^{-2}\left(1-\frac{x}{3}\right)^{-2}$. Only a small number of candidates dealt correctly with both $(-3)^{-1}$ and $(-3)^{-2}$ and went on to reach the correct final answer.

Answers: **(i)** $\frac{1}{x+1} + \frac{3}{x-3} + \frac{12}{(x-3)^2}$, **(ii)** $\frac{4}{3} - \frac{4}{9}x + \frac{4}{3}x^2$