A-Level Mathematics

Paper 1

Unsolved Topical

Past Papers with Marking Schemes

All Variants

2014-2021

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PREFACE

Excellence in learning cannot be claimed without application of concepts in a dexterous way. In this regard one of the logical approach is to start in chunks; like chapter wise learning and applying the concept on exam based questions.

This booklet provides an opportunity to candidates to practice topic wise questions from previous years to the latest. Extensive working of Team MS Books has tried to take this booklet to perfection by collaborating with top of the line teachers.

We have added answer key / marks scheme at the end of each topic for the candidate to compare the his/her answer to the best.

MS Books strives to maintain actual spacing between consecutive questions and within options as per CAIE format which gives students a more realistic feel of attempting question.

Review, feedback and contribution in this booklet by various competent teachers of a subject belonging to renowned school chains make it most valuable resource and tool for both teachers and students.
With all belief in strength of this resource material I can confidently claim that it is worth in achieving brilliance.

Our sincere thanks and gratification to Mr. Zafar Iqbal who took out special time to help compile and manage this booklet. We would also like to appreciate Mathematics faculty for reviewing and indorsing it.

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1 **(i)** Express $4x^2 - 12x$ in the form $(2x + a)^2 + b$. [2]

(ii) Hence, or otherwise, find the set of values of x satisfying $4x^2 - 12x > 7$. [2]

Q8/13/M/J/14

2 **(i)** Express $2x^2 - 10x + 8$ in the form $a(x + b)^2 + c$, where a, b and c are constants, and use your answer to state the minimum value of $2x^2 - 10x + 8$. [4]

(ii) Find the set of values of k for which the equation $2x^2 - 10x + 8 = kx$ has no real roots. [4]

Q5/11/O/N/14

3 Find the set of values of k for which the line $y = 2x - k$ meets the curve $y = x^2 + kx - 2$ at two distinct points. [5]

Q3/13/O/N/14

4 **(i)** Express $9x^2 - 12x + 5$ in the form $(ax + b)^2 + c$. [3]

(ii) Determine whether $3x^3 - 6x^2 + 5x - 12$ is an increasing function, a decreasing function or neither. [3]

Q1/13/M/J/15

5 Express $2x^2 - 12x + 7$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

Q6/11/O/N/15

6 A curve has equation $y = x^2 - x + 3$ and a line has equation $y = 3x + a$, where a is a constant.

(i) Show that the x-coordinates of the points of intersection of the line and the curve are given by the equation $x^2 - 4x + (3 - a) = 0$. [1]

(ii) For the case where the line intersects the curve at two points, it is given that the x-coordinate of one of the points of intersection is -1. Find the x-coordinate of the other point of intersection. [2]

(iii) For the case where the line is a tangent to the curve at a point P, find the value of a and the coordinates of P. [4]

Q1/13/O/N/15

7 A line has equation $y = 2x - 7$ and a curve has equation $y = x^2 - 4x + c$, where c is a constant. Find the set of possible values of c for which the line does not intersect the curve. [3]

Q6/11/M/J/16

8 **(a)** Find the values of the constant m for which the line $y = mx$ is a tangent to the curve $y = 2x^2 - 4x + 8$. [3]

(b) The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 + ax + b$, where a and b are constants. The solutions of the equation $f(x) = 0$ are $x = 1$ and $x = 9$. Find

(i) the values of a and b, [2]

(ii) the coordinates of the vertex of the curve $y = f(x)$. [2]

Q1/11/O/N/16

9 **(i)** Express $x^2 + 6x + 2$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

(ii) Hence, or otherwise, find the set of values of x for which $x^2 + 6x + 2 > 9$. [2]

Q3/12/O/N/16

10 A curve has equation $y = 2x^2 - 6x + 5$.

(i) Find the set of values of x for which $y > 13$. [3]

(ii) Find the value of the constant k for which the line $y = 2x + k$ is a tangent to the curve. [3]

Q1/13/O/N/16

11 Find the set of values of k for which the curve $y = kx^2 - 3x$ and the line $y = x - k$ do not meet. [3]

Q3/13/M/J/17

12 Find the coordinates of the points of intersection of the curve $y = x^{\frac{2}{3}} - 1$ with the curve $y = x^{\frac{1}{3}} + 1$. [4]

Q7/12/M/J/18

13 The function f is defined by $f : x \mapsto 7 - 2x^2 - 12x$ for $x \in \mathbb{R}$.

(i) Express $7 - 2x^2 - 12x$ in the form $a - 2(x + b)^2$, where a and b are constants. [2]

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(ii) State the coordinates of the stationary point on the curve $y = f(x)$. [1]

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The function g is defined by $g : x \mapsto 7 - 2x^2 - 12x$ for $x \geqslant k$.

(iii) State the smallest value of k for which g has an inverse. [1]

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(iv) For this value of k, find $g^{-1}(x)$. [3]

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Q2/11/M/J/14

Question 1

In part **(i),** candidates experienced difficulty in dealing with the $4x^2$ term and with the required form of the answer. A significant number of candidates kept 4 outside the bracket. Very few candidates thought of putting the two expressions identically equal to each other and equating coefficients, which is an alternative way of dealing with such questions. There were two ways of approaching part **(ii)**. Candidates could have used their answer to part **(i)**, taking their value of b to the other side of the equation to obtain $(2x - 3)^2 > 16$. Taking the square root of both sides gives $2x - 3 > 4$ or $2x - 3 < -4$, from which the solutions come very easily. Alternatively, candidates could have made the given inequality into a quadratic equation, solved the equation giving two critical values and deciding whether to take the region between the two critical values or the region outside the critical values. Candidates who chose the first method often made a mistake when taking the square root and wrote $2x - 3 > \pm 4$ giving one correct solution and one incorrect solution. Candidates who employed the other method often obtained the correct critical values but did not always manage to proceed correctly to obtain the correct solution to the inequality.

Answers: **(i)** $(2x - 3)^2 - 9$; **(ii)** $x < -\frac{1}{2}$, $x > 3\frac{1}{2}$.

Q8/13/M/J/14

Question 2

Both parts of this question were well answered with many candidates achieving full marks. In part **(i)** the values of a, b and c were usually correct. For the minimum value quite a number stated the x value while others gave both x and y as a pair of coordinates. In part **(ii)** most candidates used the right method for finding the discriminant and knew that this needed to be negative. There were, however, some sign errors when squaring the coefficient of x and this usually led to the critical values from the quadratic in k being +18, +2 instead of –18, –2. Most candidates correctly chose the region inside the critical values.

Answers: **(i)** $2(x-2\frac{1}{2})^2-4\frac{1}{2}$, minimum value $-4\frac{1}{2}$; **(ii)** $-18 < k < -2$.

Q5/11/O/N/14

Question 3

This type of problem provides the opportunity for candidates to use their skills in algebraic manipulation to group like terms, select coefficients and solve inequalities. A few realised that the coefficient of x and the constant both consisted of 2 terms and then used the discriminant to find the critical values. Most solutions ended after the elimination of y. Those who realised a set of values of k was required rarely illustrated this with the correct diagram or inequalities.

Answer: $k < 2$, $k > 6$

Q3/13/O/N/14

Question 4

Only a relatively small proportion of candidates provided a fully correct response to part **(i)**, many giving the answer in the more familiar form $a(x+b)^2+c$. The answer $9(x-\frac{2}{3})^2+1$, which received partial credit, occurred at least as commonly as the correct answer. In order to gain any credit in part **(ii)** it was necessary to construct valid reasoning in order to conclude that it is an increasing function. Unfortunately, relatively few made clear the connection between part **(i)** and the derivative of the function and often proceeded, unsuccessfully, to work with the second derivative or to substitute values into the derivative.

Answers: **(i)** $(3x-2)^2+1$; **(ii)** Increasing since derivative = $(3x-2)^2+1$ which is greater than 0

Q1/13/M/J/15

Question 5

This question was very well answered with the majority of candidates scoring all 3 marks. The most common error was in finding the value of the constant term, - the error usually occurring when a factor of 2 was taken out.

Answer: $2(x-3)^2-11$.

Q6/11/O/N/15

Question 6

In **part (i)** the two expressions for *y* were usually equated and rearranged correctly to produce the given answer.

Part (ii) inspired the use of a variety of methods with many choosing to substitute $x = -1$ into the given result to find *a* and then solving the quadratic for the second value of *x*. Those who appreciated the sum of the roots of the quadratic equalled 4 arrived at the value in one step.

The form of the result in **part (i)** was there to suggest the use of the discriminant in **part (iii)** and this was often seen. Those who chose to equate the gradients of the line and curve at point P were equally successful.

Answers: **(ii)** 5 **(iii)** –1, (2,5)

Q1/13/O/N/15

Question 7

Most candidates managed to score the first two marks in this question but the final mark was frequently not scored. This was most often due to failing to reverse the inequality having reached $-4c < -8$ - giving the answer as $c < 2$. Other common errors included setting the discriminant to ' = 0 ' or even ' > 0 ' as well as errors made when removing the bracket from $36 - 4(c + 7)$. Other valid methods, not involving the use of the discriminant, were also occasionally seen.

Answer: $c > 2$.

Q6/11/M/J/16

Question 8

In part **(a)** the easiest route to the solutions for m via the discriminant was not the most favoured. Some found the gradient of the curve and substituted this for m but only those who then equated their new linear equation with the quadratic made any headway.

Part **(b)** was very well answered. In part **(i)** the values of a and b were found successfully either by using the factors $(x-1)(x-9)$ or by forming a pair of simultaneous equations.

In part **(ii)** a variety of successful methods included calculus, completing the square and using the symmetry of the function. Those who chose to use $x = \frac{-b}{2a}$ had to be careful to distinguish between the values they had found and the coefficients of the quadratic.

Answer: **(a)** $m=-12,\ m=4$ **(b)(i)** $a=-10,\ b=9$ **(ii)** $(5,\ -16)$

Q1/11/O/N/16

Question 9

Part (i) was well answered by many candidates and, for them, provided a straightforward start to the paper. It was expected that the result from **part (i)** would be used in **part (ii)** but this was rarely the case. Most preferred to restart using the quadratic equation: $x^2 + 6x - 7 = 0$. The answers –7 and 1 were often left as the final answer or attached to an incorrect inequality.

Answers: **(i)** $(x + 3)^2 - 7$ **(ii)** $x < -7$, $x > 1$

Q3/12/O/N/16

Question 10

Very many candidates obtained full marks in both parts of this question. In **part (i)** a few candidates ignored the 13 given in the question and instead attempted to solve an equation which had no real solutions. Those who did take notice of the 13 were generally able to solve the resulting inequality correctly but some errors in factorising and identifying the correct regions did occur. In **part (ii)** the most common approach was to equate the curve and the line and then to use the discriminant. This was usually done successful but some weaker candidates struggled to correctly evaluate it with $64 - 8(5 - k)$ sometimes becoming $64 - 40 - 8k$. An alternative approach was to differentiate the curve and equate it to 2, being the gradient of the line. Those who used this method were usually successful although some did equate the derivative to 0 and received no credit.

Answers: (i) $x < -1$, $x > 4$ (ii) $k = -3$

Q1/13/O/N/16

Question 11

The vast majority were able to eliminate y, rearrange into a 3-term quadratic equation and find the discriminant, earning the first 2 marks. Very many candidates failed to score the final mark, often giving the answer as $k > \pm 2$ or assuming that the correct region for k was between the critical values, giving the answer as $-2 < k < 2$. Candidates who drew a diagram as an aid to solving the inequality were generally the most successful.

Answer: $k > 2$, $k < -2$.

Q3/13/M/J/17

Question 12

Most candidates were able to eliminate y and simplify to a 3-term equation, scoring the first mark. The most successful approach from this point was to employ a dummy variable u, for example , solve the quadratic equation in u, transform the solutions back to $x^{1/3}$ and cube the solutions to find x and then y. However, a variety of errors were seen which meant that a substantial numbers of candidates scored the first mark only. Some candidates chose to use y as their dummy variable and, despite the danger of confusion, a greater proportion of these candidates were able to progress to correct solutions. Another group of candidates, from their equation in $x^{1/3}$, cubed individual terms of the equation, or cubed the whole equation. Some candidates spotted one solution ($x = 8$) but not the second solution and yet other candidates tried to use logarithms. None of these approaches scored any of the last 3 marks.

Answer : (8, 3), (–1, 0).